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7114 McCallum Street
Philadelphia, PA 19119
215 247-8784 phone
www.roofmeadow.com

Mathematical Simulation Methods A Foundation for a Developing a General-Purpose Green Roof Simulation Model

With interest increasing in the benefits of green roofs, it is becoming important to have access to reliable methods for predicting the performance of green roofs over a wide range of design types and operating conditions. The much-used SCS non-dimensionalized unit hydrograph method (i.e., TR-20)¹ is not well adapted for predicting runoff from green roofs. Using actual data, or the output from other computational methods (see below), it is possible to select values of CN, Tc, and IA that will provide realistic runoff rates over small ranges of rainfall intensity. However, this approach is problematic since it is not grounded in the actual physical processes that generate runoff from green roofs. Furthermore, it is cumbersome, prone to error, and cannot easily be reduced to a standardized methodology.

Four techniques that have been used successfully to predict the hydrologic properties of green roofs hold the most promise. These are:

1. Empirical models, that rely on a large experimental database
2. Physical models, that solve the field equations for unsaturated flow,
3. Analytical models, that treat green roofs as combination of linear storage reservoirs
4. Water-balance models, that treat green roofs as simple reservoirs with restricted outlets

Experience with each of these models show that the hydrologic response of green roofs can be varied across a broad range by manipulating the physical properties of the media and the layered structure of the systems. The following is a brief summary of the four methods that are presently in use or development.

Empirical Models

These models rely on a statistical analysis of runoff collected at experiment stations. They can provide very reliable estimates of runoff, provided the hypothetical designs are close analogues to the green roof systems in the database. One such model is the *RWS Simulationsprogramm*, developed by Optigrün International AG.² It has a long and successful track record related to projects in Germany.

The great advantage of these models is their ability to evaluate the performance of a green roof system over periods of months or years. Estimates of total runoff reduction and statistical summaries of peak runoff rates can be readily generated from the output.

Limitations include the requirement that green roof systems must be close analogues to the proposed designs and that the climatic conditions must be similar to those under which the data was collected. Also, this type of program cannot be readily integrated into conventional stormwater simulation programs. However, these models may become important as a method of

¹ U.S. Department of Agriculture, 1983, Natural Resources Conservation Service, Computer Program for Project Formulation Hydrology, Technical Release 20, Washington, D.C.

² Mann, G., 2000, Grünes Licht für grüne Dächer, Deutsches Ingenieurblatt, October

generating artificial data sets for comparison with the output from proto-type simulators based on other approaches (see below).

Two-Dimensional Unsaturated-Flow Models

Preliminary work in this area has been undertaken by Roofscapes, Inc. in Philadelphia.³ The model output was calibrated against pilot scale research data collected by Optigrün International AG and used in their *RWS Simulationsprogramm* database. Hydrus 2-D⁴ was used to numerically solve the Richards' equation for saturated-unsaturated flow in porous media. Boundary conditions can be applied that describe drainage conditions, moisture uptake by plants, and evapo-transpiration.

Sensitivity analysis of the preliminary simulations supports the assertion that the most important variables are:

- Thickness of the media layers
- Volumetric field capacity
- Initial volumetric moisture content
- Slope of the matric potential function (in combination with the field capacity, this defines the water retention properties of the media)
- Saturated hydraulic conductivity

The slope of the matric potential curve can be approximated as straight-line function connecting the field capacity and maximum water capacity (MWC). The MWC is commonly used in Germany to describe green roof media.⁵ The MWC is frequently two to three times higher than the field capacity. The difference between the field capacity and the MWC represents moisture that a green roof can store temporarily and that will be released gradually over hours or days. All the data inputs can be provided through standardized laboratory tests of prospective media. One advantage of the physical model is the capability of evaluating a wide range of drainage conditions.

The output predicts the actual pattern of two-dimensional seepage flow through the green roof. This information can be very useful in helping develop an understanding of how the geometry of the green roof structure affects performance. This type of model is most effectively used to evaluate short-term response to single-storm events. A critical disadvantage is the complexity of the model, which may make it too cumbersome to become the basis for a general-purpose subroutine. A simplified one-dimensional model based on the Richards' equation is being investigated by the Canadian National Research Council and the Institute for Environmental Studies.⁶

One-Dimensional Analytical Method

This approach treats runoff from a multi-layered green roof as a cascade from a combination of linear storage elements. Each soil layer is modeled by a separate storage element. The model assumes that the flow from each soil layer is proportional to the amount of water stored in the

³ Miller, C. and Pyke, G., 1999, *Methodology for the Design of Vegetated Roof Covers*, Proceedings of the 1999 International Water Resource Engineering Conference, ASCE, Seattle, WA

⁴ Simunek, J., et al., 1996, Simulation of Water Flow and Solute Transport in Two-Dimensional Variably Saturated Media, U.S. Salinity Laboratory, USDA, ARS, Riverside, California, IGWMC-TPS53, International Ground Water Modeling Center.

⁵ According to (FLL) Richlinien für die Planung, Ausführung and Pflege von Dachbegrünung, Forschungsgesellschaft Landschaftsentwicklung Landschaftsbau e.V.= 1995. pp.56.

⁶ Personal communications with Dr. Brad Bass, University of Toronto

layer. Zimmer and Geiger, at the University of Essen, have described this approach in detail.⁷ They have also demonstrated the viability of the technique through the analysis of both experimental and field data.

The model predicts that appreciable runoff will be delayed until the field capacity of all of the layers is satisfied. After the field capacity is satisfied, the runoff characteristics of the green roof can be approximated with the use of one parameter, the system storage coefficient, K. Generally, K would be evaluated by calibration to experimental data. Zimmer and Geiger suggest employing the Fourier transform of the rainfall-runoff time series in order to streamline the determination of storage coefficients. However, storage coefficients can also be estimated using a simple mathematical formula. The formula requires the following information about each media layer in the system: porosity, thickness, field capacity, saturated hydraulic conductivity, and slope of the matric potential function.⁸

This conceptual approach may offer the simplest approach for inserting a green roof runoff generation subroutine into existing hydrologic models. A subroutine could be developed that relies on only three inputs for each layer; field capacity, the initial volumetric moisture content, and the storage coefficient.⁹ The layer storage coefficient can be determined using standardize procedures. Using these published procedures, green roof providers could furnish storage coefficients for their various systems as part of their product specification.

This approach is computationally efficient and could be used for both short-term and long-term simulations. Plant moisture uptake and evapo-transpiration can also be added to the model.

Water-Balance Technique

This method treats the green roof system like a simple reservoir and uses a time-stepping analysis to account for incremental additions and losses (including runoff) from the system. It is easily implemented in a spreadsheet or linear program. On a crude level it can account for some of the dynamic properties of green roof hydrology. The key inputs to this model consist of:

- Field capacity
- Maximum water capacity, MWC
- Storage half-life

The storage half-life is the time it takes for ½ of the water remaining in the green roof to drain away.

The model predicts that runoff will not begin until the field capacity of the system is exceeded. After this point runoff will occur as determined by the first-order rate function. If rainfall exceeds runoff, the storage capacity of the green roof will continue to fill. When the MWC is attained runoff will equal rainfall. Evapo-transpiration can also be added to the model.

The field capacity and MWC are measurable properties of the system, while the storage half-life would have to be determined by calibration to field data. CH2M HILL, Inc. is developing a prototype model based on this approach.¹⁰ This approach offers an immediate tool for evaluating green roof runoff. However, it is probably too limited to the form the basis for a general-purpose industry simulator.

⁷ Zimmer, U., and Geiger, W.F., *Model for the Design of Multi-layered Infiltration Systems*, Wat. Sci. Tech., Vol. 36, No. 8-9, pp.301, 1997

⁸ Assuming that the matric potential function has a direct relationship to the volumetric moisture content versus unsaturated hydraulic conductivity function.

⁹ or Fourier transform function

¹⁰ Personal communication with Kim Stephens, CH2M HILL, Vancouver