Mathematical Simulation Methods:
A Foundation for a Developing General-Purpose Green Roof Simulation Models

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With interest increasing in the benefits of green roofs, it is becoming important to have access to reliable methods for predicting the performance of green roofs over a wide range of design types and operating conditions. Furthermore, it is essential that these methods be based on codes or algorithms that are in the public domain and available for researchers to adapt and improve. The absence of a recognized method to predict green roof performance is one of the primary impediments to the widespread application of green roofs in the United States. It is my opinion the mathematical groundwork for the development of efficient computer subroutines has already been laid by researchers in the US and Europe.

Stormwater Runoff Simulation

The much-used SCS non-dimensionalized unit hydrograph method (i.e., TR-20)[1] is not well adapted for predicting runoff from green roofs. Using actual data, or the output from other computational methods, it is possible to select values of CN, Tc, and IA that will provide realistic runoff rates over small ranges of rainfall intensity. However, this approach is problematic since it is not grounded in the actual physical processes that generate runoff from green roofs. Furthermore, it is cumbersome, prone to error, and cannot easily be reduced to a standardized methodology.

Four techniques that have been used successfully to predict the hydrologic properties of green roofs hold the most promise. These are:

1. Empirical models, that rely on a large experimental database
2. Physical models, that solve the field equations for unsaturated flow,
3. Analytical models, that treat green roofs as combination of linear storage reservoirs
4. Water-balance models, that treat green roofs as simple reservoirs with restricted outlets

Experience with each of these models show that the hydrologic response of green roofs can be varied across a broad range by manipulating the physical properties of the media and the layered structure of the systems. In my opinion the best candidate for developing a general-purpose runoff simulation code is the one-dimensional analytical model.

This approach treats runoff from a multi-layered green roof as a cascade from a combination of linear storage elements. Each soil layer is modeled by a separate storage element. The model assumes that the flow from each soil layer is proportional to the amount of water stored in the layer. Zimmer and Gieger, at the University of Essen, have described this approach in detail.[2] They have also demonstrated the viability of the technique through the analysis of both experimental and field data.

The model predicts that appreciable runoff will be delayed until the field capacity of all of the layers is satisfied. After the field capacity is satisfied, the runoff characteristics of the green roof can be approximated with the use of one parameter, the system storage coefficient, K. Generally, K would be evaluated by calibration to experimental data. Zimmer and Geiger suggest employing the Fourier transform of the rainfall-runoff time series in order to streamline the determination of storage

[1] U.S. Department of Agriculture, 1983, Natural Resources Conservation Service, Computer Program for Project Formulation Hydrology, Technical Release 20, Washington, D.C.

[2] Zimmer, U., and Geiger, W.F., *Model for the Design of Multi-layered Infiltration Sy*stems, Wat. Sci. Tech., Vol. 36, No. 8-9, pp.301, 1997

coefficients. However, storage coefficients can also be estimated using a simple mathematical formula. The formula requires the following information about each media layer in the system: porosity, thickness, field capacity, saturated hydraulic conductivity, and slope of the matric potential function.[3]

This conceptual approach may offer the simplest way for inserting a green roof runoff generation subroutine into existing hydrologic models. A subroutine could be developed that relies on only three inputs for each layer; field capacity, the initial volumetric moisture content, and the storage coefficient.[4] The layer storage coefficient can be determined using standardize procedures. Using these published procedures, green roof providers could furnish storage coefficients for their various systems as part of their product specification. This approach is computationally efficient and could be used for both short-term and long-term simulations. Plant moisture uptake and evapo-transpiration can also be added to the model.

In the meantime, until a general algorithm can be developed, green roof runoff can be approximated using a simple water balance approach. This method treats the green roof system like a simple reservoir and uses a time-stepping analysis to account for incremental additions and losses (including runoff) from the system. It is easily implemented in a spreadsheet or linear program. On a crude level it can account for some of the dynamic properties of green roof hydrology. The key inputs to this model consist of:

- Field capacity
- Maximum water capacity, MWC
- Storage half-life

The storage half-life is the time it takes for ½ of the water remaining in the green roof to drain away.

The model predicts that runoff will not begin until the field capacity of the system is exceeded. After this point runoff will occur as determined by the first-order rate function. If rainfall exceeds runoff, the storage capacity of the green roof will continue to fill. When the MWC is attained runoff will equal rainfall. Evapo-transpiration can also be added to the model.

The field capacity and MWC are measurable properties of the system, while the storage half-life must be determined by calibration to field data. This approach offers an immediate tool for evaluating green roof runoff. However, it is probably too limited to the form the basis for a general-purpose industry simulator.

Heat Flow

The influence of a green roof on heat flow through the underlying roof is complex. It involves numerous physical processes, all of which may be too important to ignore, if a reliable simulation of heat flow is required. These processes include:

- Shading effect (i.e., absorption of radiation by the foliage)
- Sheltering (i.e., reducing heat transfer by wind advection)
- Thermal mass effect (i.e., dynamic heat absorption (and release) by the media)
- Evapo-transpiration (i.e., evaporation from the leaves of the plants)
- Simple insulation, principally as a result of trapped or quiescent air in the green roof profile

Both the insulation and thermal mass effects contribute to a 'damping' of the response of the green roof changes in temperature. However, generally both effects are not prominent at the same time. Insulation is inversely proportional to moisture content, while heat absorption (heat capacity) is proportional.

[3] Assuming that the matric potential function has a direct relationship to the volumetric moisture content versus unsaturated hydraulic conductivity function.

[4] or Fourier transform function

Due to the complexity and interplay of all of all these factors, most data pertaining to the thermal properties of green roofs is only useful in leading to generalizations about potential benefits. Predictions of thermal energy management benefits are possible only with the assistance of building envelope analysis techniques that can integrate all of these effects.

A novel approach to this problem has been suggested by Shade Consultants.[5] They propose to represent the heat transfer components through the use of electric circuit analogs. For instance, thermal mass appears as capacitance, while insulation is represented by resistance. Heat flow is current. It is quite likely that a practical building envelope analysis technique could be developed in this way.

[5] communication with Chris Wark (11-20-02), Shade Consultants LLC, Peoria, IL